

1. Let  $G$  be any undirected graph with positive edge weights, and let  $T$  be a minimum spanning tree of  $G$ . For any two vertices,  $u$  and  $v$ , let  $d1(u, v)$  and  $d2(u, v)$  be the shortest distances between  $u$  and  $v$  in  $G$  and  $T$ , respectively. Which ONE of the options is CORRECT for all possible  $G, T, u$ , and  $v$ ?

- a.  $d1(u, v) = d2(u, v)$
- b.  $d1(u, v) \leq d2(u, v)$
- c.  $d1(u, v) \geq d2(u, v)$
- d.  $d1(u, v) \neq d2(u, v)$

2. What is the output of the following C program?

```
#include <stdio.h>
int main()
{
double a[2]={20.0, 25.0}, *p, *q;
p = a;

q = p + 1;

printf("%d,%d", (int)(q - p), (int)(*q - *p));

return 0;
}
```

- a. 4, 8
- b. 1, 5
- c. 8, 5
- d. 1, 8

3. Consider an array  $X$  that contains  $n$  positive integers. A subarray of  $X$  is defined to be a sequence of array locations with consecutive indices. The C code snippet given below has been written to compute the length of the longest subarray of  $X$  that contains at most two distinct integers. The code has two missing expressions labelled  $(P)$  and  $(Q)$ .

```
int first=0, second=0, len1=0, len2=0, maxlen=0;
```

```
for (int i=0; i < n; i++) {
```

```
if (X[i] == first) {
```

```
len2++; len1++;
```

```
} else if (X[i] == second) {
```

```

len2++;

len1 = (P) ;

second = first;

} else {

len2 = (Q) ;

len1 = 1; second = first;

}

if (len2 > maxlen) { maxlen = len2;

}

first = X[i];

}

```

Which one of the following options gives the CORRECT missing expressions?

- a. (P) len1+1 (Q) len2+1
- b. (P) 1 (Q) len1 + 1
- c. (P) 1 (Q) len2+1
- d. (P) len2+1 (Q) len1+1

4. The eigenvalues of a  $2 \times 2$  matrix are 1 and  $-1$ , and its singular values are 1 and 0. Then which of the following is (are) true?

- a. Rank of the matrix is 2.
- b. Rank of the matrix is 1.
- c. Rank of the matrix is 0.
- d. Such a matrix cannot exist.

5. Two sets of numbers are given as sorted arrays in increasing order, A and B, with n and m numbers respectively. What is the best estimate for the complexity of computing the set  $A \setminus B$ .

- a.  $O(n.m)$

- b.  $O(n^2 \cdot m)$
- c.  $O(n + m)$
- d.  $(n \log n + m \log m)$

6. There is a height-balanced tree  $T$  storing  $n$  elements. The greatest element smaller than  $x$  in  $T$  is called the predecessor of  $s$  in  $T$ . Given any positive number  $k$ , we wish to compute  $k$  predecessors of  $x$  in  $T$ . What is the time complexity of the best algorithm that can do this task?

- a.  $O(k + \log n)$
- b.  $O(k \log n)$
- c.  $O(\log(n + k))$
- d.  $O(n + k)$

7. Suppose we are given  $n$  keys,  $m$  hash table slots, and two simple uniform hash functions  $h_1$  and  $h_2$ . Further suppose our hashing scheme uses  $h_1$  for the odd keys and  $h_2$  for the even keys. What is the expected number of keys in a slot?

- a.  $m/n$
- b.  $n/m$
- c.  $2n/m$
- d.  $n/2m$

8. Consider a special queue called MirrorQueue that supports three operations:

- Enqueue( $x$ ) - inserts an element at the rear
- Dequeue()- removes an element from the front
- Front() - returns the front element without removing it.

Initially, MirrorQueue contains elements [1,2,3,4,5] (front=1)

We want to **print all elements twice** in the following order:

1,2,3,4,5,5,4,3,2,1

so that the **final content of the queue remains unchanged**.

You can only use the three supported operations (Enqueue, Dequeue, and Front).

What is the **minimum total number of operation calls** required?

9. Consider a Depth First Search (DFS) traversal and the *start time* and *finish time* assigned to vertices during DFS.

- a. In a DFS of a directed graph, it is possible to have an edge between two vertices  $u$  and  $v$  such that  $u$  has start and finish times  $(10,40)$  and  $v$  has start and finish times  $(30,50)$ .
- b. In a DFS of an undirected graph, it is possible to have an edge between two vertices  $u$  and  $v$  such that  $u$  has start and finish times  $(5,20)$  and  $v$  has start and finish times  $(30,50)$ .
- c. In a DFS of a directed graph, it is possible to have an edge between two vertices  $u$  and  $v$  such that  $u$  has start and finish times  $(5,20)$  and  $v$  has start and finish times  $(30,50)$ .
- d. In a DFS of a directed graph, if vertex  $u$  has start and finish times  $(10,20)$  and vertex  $v$  has start and finish times  $(15,18)$ , then the edge  $(v,u)$  must be a back edge.

10. The symbol " $\rightarrow$ " indicates functional dependency in the context of a relational database. Which of the following options is/are TRUE?

- a.  $(X,Y) \rightarrow (Z,W)$  implies  $X \rightarrow (Z,W)$
- b.  $(X,Y) \rightarrow (Z,W)$  implies  $(X,Y) \rightarrow Z$
- c.  $((X,Y) \rightarrow Z$  and  $W \rightarrow Y)$  implies  $(X, W) \rightarrow Z$
- d.  $(X \rightarrow Y$  and  $Y \rightarrow Z)$  implies  $X \rightarrow Z$ .